Goldstein 2.10.

$$y = at + bt^2 , \qquad \dot{y} = a + 2bt$$

$$\Rightarrow \quad T = \frac{m}{2}(a+2bt)^2 , \quad V = mg(at + bt^2)$$

$$\Rightarrow \quad L = \frac{m}{2}(a+2bt)^2 - mg(at + bt^2)$$

Clearly, we have taken a, b as coordinates.
The constraint is:

$$a\sqrt{\frac{2y_0}{g}} + b\left(\frac{2y_0}{g}\right) - y_0 = 0 .$$

Then

$$\delta\int_{t_0}^{t_1}\left\{L + \lambda(a,b,t)\left[a\sqrt{\frac{2y_0}{g}} + b\left(\frac{2y_0}{g}\right) - y_0\right]\right\}dt = 0$$

This suggests we look for eqm in the form of Lagr with Lagr multipliers considered. Since $\dot{a}$, $\dot{b}$ don't appear in the Lagr, we clearly have eqms:

$$\begin{cases} \frac{\partial L}{\partial a} = 0 = m(a+2bt) - mgt + \lambda\sqrt{\frac{2y_0}{g}} \\ \frac{\partial L}{\partial b} = 0 = m(a+2bt)(2t) - mgt^2 + \lambda\left(\frac{2y_0}{g}\right) \\ a\sqrt{\frac{2y_0}{g}} + b\left(\frac{2y_0}{g}\right) - y_0 = 0 \end{cases}$$

3 equations, for 3 coordinates a, b, λ.

The constraint equation gives

$$a = \left[y_0 - b\left(\frac{2y_0}{g}\right) \right] \Big/ \sqrt{\frac{2y_0}{g}}$$

$$= y_0 \sqrt{\frac{g}{2y_0}} - b\sqrt{\frac{2y_0}{g}}$$

At $t=0$, the other two equations give

$$\begin{cases} ma + \lambda \sqrt{\frac{2y_0}{g}} = 0 \\ \lambda \left(\frac{2y_0}{g}\right) = 0 \end{cases}$$

$$\Rightarrow \boxed{a = 0}$$

Solving for b: $b\sqrt{\frac{2y_0}{g}} = y_0 \sqrt{\frac{g}{2y_0}}$

$$b = \cancel{y_0} \frac{g}{2\cancel{y_0}}$$

$$= \boxed{\frac{g}{2}}$$

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